

## 1.11 Exercises

1–12 ■ Write an equation that expresses the statement.

- $T$  varies directly as  $x$ .
- $P$  is directly proportional to  $w$ .
- $v$  is inversely proportional to  $z$ .
- $w$  is jointly proportional to  $m$  and  $n$ .
- $y$  is proportional to  $s$  and inversely proportional to  $t$ .
- $P$  varies inversely as  $T$ .
- $z$  is proportional to the square root of  $y$ .
- $A$  is proportional to the square of  $t$  and inversely proportional to the cube of  $x$ .
- $V$  is jointly proportional to  $l$ ,  $w$ , and  $h$ .
- $S$  is jointly proportional to the squares of  $r$  and  $\theta$ .
- $R$  is proportional to  $i$  and inversely proportional to  $P$  and  $t$ .
- $A$  is jointly proportional to the square roots of  $x$  and  $y$ .

13–22 ■ Express the statement as an equation. Use the given information to find the constant of proportionality.

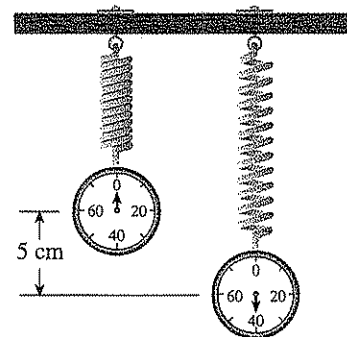
- $y$  is directly proportional to  $x$ . If  $x = 6$ , then  $y = 42$ .
- $z$  varies inversely as  $t$ . If  $t = 3$ , then  $z = 5$ .
- $M$  varies directly as  $x$  and inversely as  $y$ . If  $x = 2$  and  $y = 6$ , then  $M = 5$ .
- $S$  varies jointly as  $p$  and  $q$ . If  $p = 4$  and  $q = 5$ , then  $S = 180$ .
- $W$  is inversely proportional to the square of  $r$ . If  $r = 6$ , then  $W = 10$ .
- $t$  is jointly proportional to  $x$  and  $y$  and inversely proportional to  $r$ . If  $x = 2$ ,  $y = 3$ , and  $r = 12$ , then  $t = 25$ .
- $C$  is jointly proportional to  $l$ ,  $w$ , and  $h$ . If  $l = w = h = 2$ , then  $C = 128$ .
- $H$  is jointly proportional to the squares of  $l$  and  $w$ . If  $l = 2$  and  $w = \frac{1}{3}$ , then  $H = 36$ .
- $s$  is inversely proportional to the square root of  $t$ . If  $s = 100$ , then  $t = 25$ .
- $M$  is jointly proportional to  $a$ ,  $b$ , and  $c$ , and inversely proportional to  $d$ . If  $a$  and  $d$  have the same value, and if  $b$  and  $c$  are both 2, then  $M = 128$ .

### Applications

23. **Hooke's Law** Hooke's Law states that the force needed to keep a spring stretched  $x$  units beyond its natural length is directly proportional to  $x$ . Here the constant of proportional-

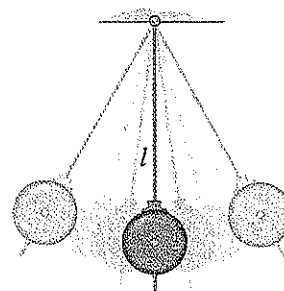
ity is called the **spring constant**.

- Write Hooke's Law as an equation.
- If a spring has a natural length of 10 cm and a force of 40 N is required to maintain the spring stretched to a length of 15 cm, find the spring constant.
- What force is needed to keep the spring stretched to a length of 14 cm?



24. **Law of the Pendulum** The period of a pendulum (the time elapsed during one complete swing of the pendulum) varies directly with the square root of the length of the pendulum.

- Express this relationship by writing an equation.
- In order to double the period, how would we have to change the length  $l$ ?



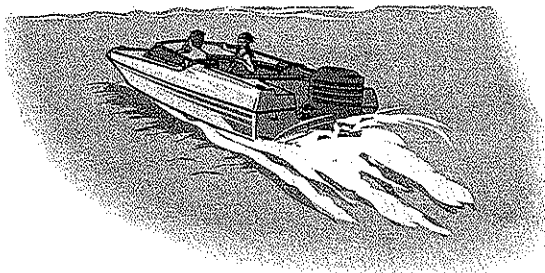
25. **Printing Costs** The cost  $C$  of printing a magazine is jointly proportional to the number of pages  $p$  in the magazine and the number of magazines printed  $m$ .

- Write an equation that expresses this joint variation.
- Find the constant of proportionality if the printing cost is \$60,000 for 4000 copies of a 120-page magazine.
- How much would the printing cost be for 5000 copies of a 92-page magazine?

26. **Boyle's Law** The pressure  $P$  of a sample of gas is directly proportional to the temperature  $T$  and inversely proportional to the volume  $V$ .
- Write an equation that expresses this variation.
  - Find the constant of proportionality if 100 L of gas exerts a pressure of 33.2 kPa at a temperature of 400 K (absolute temperature measured on the Kelvin scale).
  - If the temperature is increased to 500 K and the volume is decreased to 80 L, what is the pressure of the gas?

27. **Power from a Windmill** The power  $P$  that can be obtained from a windmill is directly proportional to the cube of the wind speed  $s$ .
- Write an equation that expresses this variation.
  - Find the constant of proportionality for a windmill that produces 96 watts of power when the wind is blowing at 20 mi/h.
  - How much power will this windmill produce if the wind speed increases to 30 mi/h?

28. **Power Needed to Propel a Boat** The power  $P$  (measured in horse power, hp) needed to propel a boat is directly proportional to the cube of the speed  $s$ . An 80-hp engine is needed to propel a certain boat at 10 knots. Find the power needed to drive the boat at 15 knots.



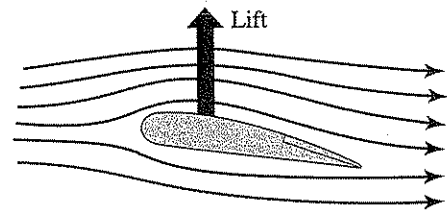
29. **Loudness of Sound** The loudness  $L$  of a sound (measured in decibels, dB) is inversely proportional to the square of the distance  $d$  from the source of the sound. A person 10 ft from a lawn mower experiences a sound level of 70 dB; how loud is the lawn mower when the person is 100 ft away?
30. **Stopping Distance** The stopping distance  $D$  of a car after the brakes have been applied varies directly as the square of the speed  $s$ . A certain car traveling at 50 mi/h can

stop in 240 ft. What is the maximum speed it can be traveling if it needs to stop in 160 ft?

31. **A Jet of Water** The power  $P$  of a jet of water is jointly proportional to the cross-sectional area  $A$  of the jet and to the cube of the velocity  $v$ . If the velocity is doubled and the cross-sectional area is halved, by what factor will the power increase?



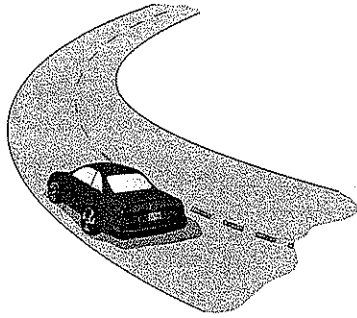
32. **Aerodynamic Lift** The lift  $L$  on an airplane wing at take-off varies jointly as the square of the speed  $s$  of the plane and the area  $A$  of its wings. A plane with a wing area of 500 ft<sup>2</sup> traveling at 50 mi/h experiences a lift of 1700 lb. How much lift would a plane with a wing area of 600 ft<sup>2</sup> traveling at 40 mi/h experience?



33. **Drag Force on a Boat** The drag force  $F$  on a boat is jointly proportional to the wetted surface area  $A$  on the hull and the square of the speed  $s$  of the boat. A boat experiences a drag force of 220 lb when traveling at 5 mi/h with a wetted surface area of 40 ft<sup>2</sup>. How fast must a boat be traveling if it has 28 ft<sup>2</sup> of wetted surface area and is experiencing a drag force of 175 lb?
34. **Skidding in a Curve** A car is traveling on a curve that forms a circular arc. The force  $F$  needed to keep the car from skidding is jointly proportional to the weight  $w$  of the car and the square of its speed  $s$ , and is inversely proportional to the radius  $r$  of the curve.
- Write an equation that expresses this variation.
  - A car weighing 1600 lb travels around a curve at 60 mi/h. The next car to round this curve weighs 2500 lb

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and requires the same force as the first car to keep from skidding. How fast is the second car traveling?



**35. Electrical Resistance** The resistance  $R$  of a wire varies directly as its length  $L$  and inversely as the square of its diameter  $d$ .

- (a) Write an equation that expresses this joint variation.
- (b) Find the constant of proportionality if a wire 1.2 m long and 0.005 m in diameter has a resistance of 140 ohms.
- (c) Find the resistance of a wire made of the same material that is 3 m long and has a diameter of 0.008 m.

**36. Kepler's Third Law** Kepler's Third Law of planetary motion states that the square of the period  $T$  of a planet (the time it takes for the planet to make a complete revolution about the sun) is directly proportional to the cube of its average distance  $d$  from the sun.

- (a) Express Kepler's Third Law as an equation.
- (b) Find the constant of proportionality by using the fact that for our planet the period is about 365 days and the average distance is about 93 million miles.
- (c) The planet Neptune is about  $2.79 \times 10^9$  mi from the sun. Find the period of Neptune.

**37. Radiation Energy** The total radiation energy  $E$  emitted by a heated surface per unit area varies as the fourth power of its absolute temperature  $T$ . The temperature is 6000 K at the surface of the sun and 300 K at the surface of the earth.

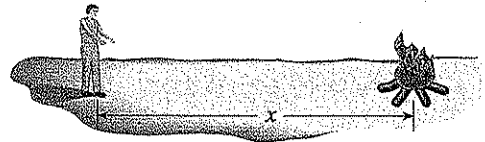
- (a) How many times more radiation energy per unit area is produced by the sun than by the earth?
- (b) The radius of the earth is 3960 mi and the radius of the sun is 435,000 mi. How many times more total radiation does the sun emit than the earth?

**38. Value of a Lot** The value of a building lot on Galiano Island is jointly proportional to its area and the quantity of water produced by a well on the property. A 200 ft by 300 ft lot has a well producing 10 gallons of water per minute, and is valued at \$48,000. What is the value of a 400 ft by 400 ft lot if the well on the lot produces 4 gallons of water per minute?

**39. Growing Cabbages** In the short growing season of the Canadian arctic territory of Nunavut, some gardeners find it

possible to grow gigantic cabbages in the midnight sun. Assume that the final size of a cabbage is proportional to the amount of nutrients it receives, and inversely proportional to the number of other cabbages surrounding it. A cabbage that received 20 oz of nutrients and had 12 other cabbages around it grew to 30 lb. What size would it grow to if it received 10 oz of nutrients and had only 5 cabbage "neighbors"?

**40. Heat of a Campfire** The heat experienced by a hiker at a campfire is proportional to the amount of wood on the fire, and inversely proportional to the cube of his distance from the fire. If he is 20 ft from the fire, and someone doubles the amount of wood burning, how far from the fire would he have to be so that he feels the same heat as before?



**41. Frequency of Vibration** The frequency  $f$  of vibration of a violin string is inversely proportional to its length  $L$ . The constant of proportionality  $k$  is positive and depends on the tension and density of the string.

- (a) Write an equation that represents this variation.
- (b) What effect does doubling the length of the string have on the frequency of its vibration?

**42. Spread of a Disease** The rate  $r$  at which a disease spreads in a population of size  $P$  is jointly proportional to the number  $x$  of infected people and the number  $P - x$  who are not infected. An infection erupts in a small town with population  $P = 5000$ .

- (a) Write an equation that expresses  $r$  as a function of  $x$ .
- (b) Compare the rate of spread of this infection when 10 people are infected to the rate of spread when 1000 people are infected. Which rate is larger? By what factor?
- (c) Calculate the rate of spread when the entire population is infected. Why does this answer make intuitive sense?

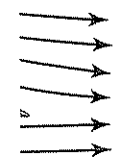
### Discovery • Discussion

**43. Is Proportionality Everything?** A great many laws of physics and chemistry are expressible as proportionalities. Give at least one example of a function that occurs in the sciences that is *not* a proportionality.

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